Two parameters will be allowed to vary, namely m_1 (equivalent to θ_1) and k_2 (equivalent to θ_2). The mass and stiffness matrices and their nonzero derivatives are

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{kg} \qquad K = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \text{N m}^{-1}$$

$$\frac{\partial M}{\partial m_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \frac{\partial K}{\partial k_2} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The first eigenvalue and mass-normalized eigenvector are

$$\lambda_1 = 1 \text{rad}^2\,\text{s}^{-2} \qquad \phi_1 = \frac{1}{\sqrt{6}} \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix} \text{kg}^{-0.5}$$

The eigenvalue derivatives may be calculated from Eq. (4) as

$$\frac{\partial \lambda_1}{\partial m_1} = -\frac{1}{6} \text{rad}^2\,\text{s}^{-2}\,\text{kg}^{-1} \qquad \frac{\partial \lambda_1}{\partial k_2} = \frac{1}{6} \text{rad}^2\,\text{s}^{-2}\,\text{N}^{-1}\text{m}$$

The eigenvector derivatives may be calculated using Eqs. (5–9) as

$$\frac{\partial \phi_1}{\partial m_1} = \frac{1}{18\sqrt{6}} \begin{Bmatrix} 5 \\ -5 \\ -4 \end{Bmatrix} \text{kg}^{-1.5}$$

$$\frac{\partial \phi_1}{\partial k_2} = \frac{1}{36\sqrt{6}} \begin{Bmatrix} 17 \\ -8 \\ -1 \end{Bmatrix} \text{kg}^{-0.5}\,\text{N}^{-1}\,\text{m}$$

The second-order eigenvalue derivatives may then be calculated from Eq. (11) as

$$\frac{\partial^2 \lambda_1}{\partial m_1^2} = -\frac{7}{108} \text{rad}^2\,\text{s}^{-2}\,\text{kg}^{-2} \qquad \frac{\partial^2 \lambda_1}{\partial k_2^2} = -\frac{25}{108} \text{rad}^2\,\text{s}^{-2}\,\text{N}^{-2}\text{m}^2$$

$$\frac{\partial^2 \lambda_1}{\partial m_1\,\partial k_2} = -\frac{5}{27} \text{rad}^2\,\text{s}^{-2}\,\text{kg}^{-1}\,\text{N}^{-1}\,\text{m}$$

Some of the intermediate steps in the calculation of the second-order eigenvector derivative with respect to m_1 will now be shown. The vector v_{111} is given by Eq. (14) as the solution of

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} v_{111} = \frac{1}{108\sqrt{6}} \begin{Bmatrix} 7 \\ -4 \\ 1 \end{Bmatrix}$$

Since the largest element of the eigenvector is the second, we will let the second element of v_{111} equal zero. Thus v_{111} is given by the solution of

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} v_{111} = \frac{1}{108\sqrt{6}} \begin{Bmatrix} 7 \\ 0 \\ 1 \end{Bmatrix} \quad \text{or} \quad v_{111} = \frac{1}{216\sqrt{6}} \begin{Bmatrix} 7 \\ 0 \\ 1 \end{Bmatrix}$$

The constant c_{111} is given by Eq. (16) as $c_{111} = -43/324$ and the eigenvector derivative by Eq. (13) as

$$\frac{\partial^2 \phi_1}{\partial m_1^2} = -\frac{1}{648\sqrt{6}} \begin{Bmatrix} 65 \\ 172 \\ 83 \end{Bmatrix} \text{kg}^{-2.5}$$

Similarly the other two-second-order derivatives are

$$\frac{\partial^2 \phi_1}{\partial k_2^2} = \frac{1}{1296\sqrt{6}} \begin{Bmatrix} 823 \\ -250 \\ 31 \end{Bmatrix} \text{kg}^{-0.5}\,\text{N}^{-2}\,\text{m}^2$$

$$\frac{\partial^2 \phi_1}{\partial m_1\,\partial k_2} = \frac{1}{648\sqrt{6}} \begin{Bmatrix} 97 \\ 107 \\ 124 \end{Bmatrix} \text{kg}^{-1.5}\,\text{N}^{-1}\,\text{m}$$

Conclusion

This Note has outlined the extension of Nelson's method to enable the calculation of second- and higher order eigenvector sensitivities. Nelson's method has the advantage that only the eigenvalue and eigenvector of interest and their lower order derivatives are required to calculate the second or higher derivatives. Furthermore the coefficient matrix required for all the derivatives of a single eigenvector are the same, with the attendant saving in computation. The method is easily extended to nonsymmetric matrices, although both left and right eigenvectors would have to be calculated.

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U-Parameter Robust Flight Control Design

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Introduction

It is well known that all compensators that robustly stabilize a plant with respect to frequency-domain perturbations can be parametrized in terms of an arbitrary bounded real function $U(s)$,[1] which is referred to as U-parameter design. This parameter has been used for the robust control design with nominal system optimization.[2] Wei et al.[3] have explored the application of U-parameter feedback design to a terrain-following flight control system.

In this Note, by applying the U parameter, we explore an application method to achieve the desired performance and robustness of the flight control system. The closed-loop transfer function of the feedback system can be formulated with free parameters. For

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given specification, we can determine the desired reference model. By a model-matching approach, we can minimize the H^2 norm of the transfer function error between the designed closed-loop system and the reference model to determine these free parameters such that the desired performance and robustness of the system can both be met. The longitudinal mode of the F-4E flight control system is considered as a design example to illustrate the validity of the application method.

U-Parameter Robust Stabilization

Consider the single-input, single-output feedback system shown in Fig. 1, where $P(s)$ and $C(s)$ denote the transfer functions of the plant and compensator, respectively. Assume that the perturbed plant $P(s)$ satisfies 1) that $P(s)$ has the same number of unstable poles as that of $P_0(s)$ and 2) that the plant uncertainty satisfies the inequality

$$|P(jw) - P_0(jw)| \le |r(jw)|, \qquad |r(jw)| > 0, \qquad \forall w \tag{1}$$

where $P_0(s)$ denotes the nominal plant and $r(s)$ is a stable minimum-phase proper rational function that characterizes the bound of the uncertainty of the plant.

Lemma 1. The closed-loop system shown in Fig. 1 is stable for all perturbed plants if and only if the closed-loop system is stable for $P(s) = P_0(s)$ and

$$|C(1 + P_0C)^{-1}||r(j\omega)| \le 0, \qquad \forall w \tag{2}$$

Definition 1. A function $U(s)$ analytic in $\mathrm{Re}[s] \ge 0$ satisfying

$$|U(jw)| \le 1, \qquad \forall w \tag{3}$$

is called bounded real (BR).

Define

$$B_p(s) = \prod_i \left\{ \frac{\alpha_i - s}{\bar{\alpha}_i + s} \right\} \tag{4}$$

$$\tilde{P}_0(s) = B_p(s)P_0(s) \tag{5}$$

$$\beta_i = \frac{r(\alpha_i)}{\tilde{P}_0(\alpha_i)} \tag{6}$$

where $B_p(s)$ is the Blaschke product, α_i denote the unstable poles of $P_0(s)$, and $\bar{\alpha}_i$ are the complex conjugate of α_i. Then we have the following theorem.

Theorem 1.[1,2] For the system shown in Fig. 1, the closed-loop system with transfer function of the form

$$T(s) = \tilde{P}_0(s)\frac{U_1(s)}{r(s)} \tag{7}$$

is stable for all perturbed plants $P(s)$, where $U_1(s)$ is a BR function that interpolates to the points $U_1(\alpha_i) = \beta_i$.

It is well known that all BR functions $U_1(s)$ that interpolate given points in $\mathrm{Re}[s] > 0$ can be parametrized in terms of an arbitrary BR function $U(s)$ such that $U_1(s)$ can be written as

$$U_1(s) = \frac{F_1(s)U(s) + F_2(s)}{F_3(s)U(s) + F_4(s)} \tag{8}$$

where the functions $F_1(s)$, $F_2(s)$, $F_3(s)$, and $F_4(s)$ are functionals of α_i and β_i.[2]

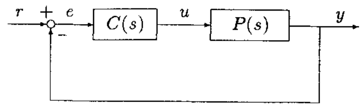

Fig. 1 Feedback control system.

F-4E Robust Flight Control System Design

In this section, the longitudinal mode of the F-4E flight control system is considered as a design example.[4] The transfer function forms of $q(s)/\delta_e(s)$ (pitch rate/deviation of elevator deflection) for four typical flight conditions can be obtained as follows:

$$\begin{aligned} \mathrm{FC1}: P_1(s) &= \frac{-13.239(s + 0.884)}{(s + 3.068)(s - 1.228)} \\ \mathrm{FC2}: P_2(s) &= \frac{-36.269(s + 1.554)}{(s + 4.904)(s - 1.784)} \end{aligned} \tag{9}$$

$$\begin{aligned} \mathrm{FC3}: P_3(s) &= \frac{-11.308(s + 0.637)}{(s + 1.878)(s - 0.560)} \\ \mathrm{FC4}: P_4(s) &= \frac{-12.320(s + 0.821)}{(s + 1.923)(s - 0.640)} \end{aligned} \tag{10}$$

Here we take the average values of poles, zeros, and steady-state gains of these flight conditions as the nominal plant

$$P_0(s) = \frac{-19.576(s + 0.974)}{(s + 2.943)(s - 1.053)} \tag{11}$$

and then we can determine the plant uncertainties for different flight conditions as shown in Fig. 2. The bound of the uncertainties is chosen as

$$r(s) = \frac{16}{s + 5} \tag{12}$$

Since $\alpha_1 = 1.053$ [unstable pole of $P_0(s)$], we have

$$\tilde{P}_0(s) = \frac{19.576(s + 0.974)}{(s + 2.943)(s + 1.053)} \tag{13}$$

and

$$\beta = \frac{r(1.053)}{\tilde{P}_0(1.053)} = 0.56 \tag{14}$$

Choosing

$$U(s) = \frac{as^2 + bs + c}{(s + d)^2} \tag{15}$$

we have

$$T(s) = \frac{19.576(s + 0.974)(s + 5)}{16(s + 2.943)(s + 1.053)} \times \frac{(s - 1.053)[(as^2 + bs + c)/(s + d)^2] + 0.56(s + 1.053)}{s + 1.053 + 0.56(s - 1.053)[(as^2 + bs + c)/(s + d)^2]} \tag{16}$$

In Eq. (16), for achieving zero steady-state error to a step input, i.e., $T(s)|_{s=0} = 1$, we obtain $c = 0.0572d^2$. Further, to ensure a proper $C(s)$, $T(s)$ has to be strictly proper; thus we have $a = -0.56$. Thus the closed-loop transfer function $T(s)$ is reduced to

$$T(s) = \frac{(2.1{+}1.78b{+}1.99d)s^4{+}(12.5{+}8.75b{+}104.9d{+}1.1d^2)s^3{+}(10.2{-}2.53b{+}22.2d{+}7.5d^2)s^2{+}({-}100b{+}10.22d{+}10.98d^2)s{+}4.59d^2}{s^5{+}(6{+}0.82b{+}2.9d)s^4{+}(11.14{+}2.4b{+}14.7d{+}1.5d^2)s^3{+}(6.26{-}0.9b{+}21.3d{+}7.48d^2)s^2{+}({-}2.65b{+}9.49d{+}10.57d^2)s{+}4.59d^2} \tag{17}$$

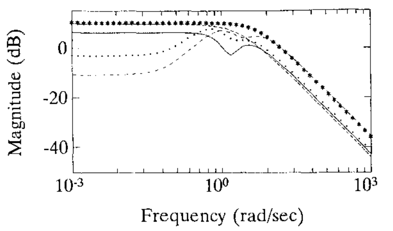

Fig. 2 Magnitude responses of uncertainties: $P_0(s) - P_1(s)$ (solid line); $P_0(s) - P_2(s)$ (dashed line); $P_0(s) - P_3(s)$ (dotted line); $P_0(s) - P_4(s)$ (dash-dotted line); uncertainty bound $r(s)$ (star line).

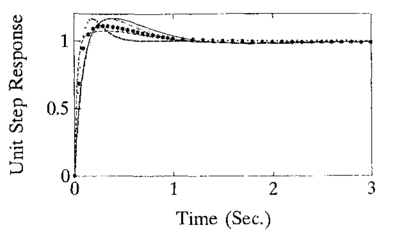

Fig. 3 Unit step responses: FC1 (solid line); FC2 (dashed line); FC3 (small-dotted line); FC4 (dash-dotted line); nominal system (star line); reference model (big-dotted line).

Robust Compensator Design with Desired Performance Requirements

In this section, the system performance will be considered. Here we use the model-matching approach to achieve the desired performance specification. It is seen that the order difference between the denominator and numerator of $T(s)$ in Eq. (17) is 1; the reference model is chosen as

$$T_m(s) = \frac{w_{sp}^2}{z_m} \frac{s + z_m}{s^2 + 2\zeta_{sp} w_{sp} s + w_{sp}^2} \tag{18}$$

Following the design specification,[4] ζ_{sp} and w_{sp} are chosen as 0.9 and 11, respectively. By adjusting the zero location $-z_m$ by the model pole-zero pattern,[5] it is found that, when $z_m = 6$, the peak time is 0.19 s and maximum overshoot is 16.5%. Thus the zero is chosen at -6; i.e., we have the reference model

$$T_m(s) = \frac{20.17(s + 6)}{s^2 + 19.8s + 121} \tag{19}$$

To minimize $\|T(s) - T_m(s)\|_2$ under the restriction that $U(s)$ is BR, for simplicity, let $b = \frac{1}{2}d$ such that $U(s)$ is always BR for all $d > 0$. By minimizing $\|T(s) - T_m(s)\|_2$ with respect to d, we have $d = 6.7$ and $b = 3.35$. Then, from Eq. (17), $T(s)$ is determined and the compensator will be

$$C(s) = \frac{-(1.1s^4 + 11.65s^3 + 41.6s^2 + 60.36s + 31.9)}{s(s^3 + 7.86s^2 + 8.06s + 1.78)} \tag{20}$$

By applying this compensator to different flight conditions, the simulation result is given in Fig. 3, which shows that this parametrized robust compensator can achieve satisfactory performance and robustness of this flight control system.

Conclusions

By using the U-parameter design method and model-matching approach, we have explored a simple parametrized robust compensator design method that shows satisfactory system performance and robustness for a flight control system through simulation. For the considered design example, the Barmish/Wei theory[6] and proportional-plus-integral controller could simultaneously stabilize the four given plants; however the U-parameter approach possesses the parametrized formulation of a simultaneous stabilization problem such that it can be easily applied for model-matching design to achieve the desired system performance.

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Methodology for Integration of Digital Control Loaders in Aircraft Simulators

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I. Introduction

It is known and accepted that frequencies up to 50 Hz must be represented in simulating control feel. This translates into a frame rate of 500 Hz or more for digital electronics.[1] Only recently have moderately priced digital systems been able to achieve this performance. The control loader electronics in use today are still predominantly analog.

The analog computer of a control loader typically represents a generic quasilinear model of a control system.[1] The parameters are selected to suit the aircraft being simulated and adjusted to fit control system responses measured in flight or on the ground.[2–4] As delivered, the new digital loaders[5] are programmed to emulate the logic wired into their analog predecessors. The digital systems offer improved repeatability and reliability while delivering equivalent functionality. The implementation reported here exploits the digital system to model the actual linkage. The specific contributions made were 1) a commanded force equation derived from the difference of the aircraft and simulator dynamic equations, 2) a recursive formalism for treating the linkage, the mathematical formulation is by induction, which feeds into recursive code. (see Ref. 6 for details), and 3) a generic force loop to control the actuator and deliver the commanded force regardless of the control position and velocity.

A trial implementation was made at the University of Alabama Flight Dynamics Laboratory (UA FDL). A force feel system was integrated for the longitudinal cyclic in the UH1 fixed base simulator using a McFadden 192B[5] digital control loader.

Our purpose was to model the control system as accurately as the hardware of the digital loader allowed (without assist from the simulation host and/or other computers). Lack of access to a simulator that requires and supports extreme fidelity, or to a flying UH1, limits the scope of our validation to checking that the implementation was

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